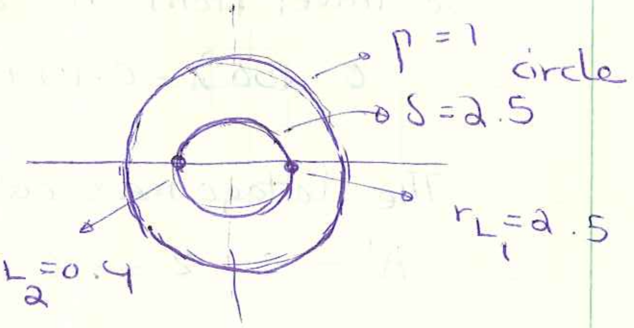
HW5 Solutions

2-37

$Z_L = 100\ \Omega$

$S = 2.5$

$Z_0 = ?$



$$z_L = \frac{Z_L}{Z_0} \Rightarrow Z_0 = \frac{Z_L}{z_L}$$

$$\rightarrow \begin{cases} Z_0 = \dfrac{100}{0.4} = 250\ \Omega \\ \text{or} \\ Z_0 = \dfrac{100}{2.5} = 40\ \Omega \end{cases}$$

2-38

$Z_0 = 50\ \Omega$ lossless line

load $Z_L = 50 + j50\ \Omega$ $\Rightarrow z_L = 1 + 0.5j$ (A)

$$|\Gamma| = \frac{18.5\ \text{mm}}{78\ \text{mm}} = 0.237$$

$\theta_r = 76.3°$ (B) $\rightarrow \Gamma = 0.23 e^{j76.3}$

$S = 1.62$ (A′)

at B $\rightarrow$ $0.144\lambda \xrightarrow{\text{thus}} 0.35\lambda$ toward the generator $\Rightarrow$

$$\begin{array}{r} 0.144\lambda \\ 0.35\lambda \\ \hline 0.494\lambda \end{array}$$

(C)

at (A″) $z_{in} = 0.61 - j0.02$

$Z_{in} = Z_0 \times z_{in} \Rightarrow Z_{in} = 30.5 - j1.0\ \Omega$

for y_{in} opposite point from z_{in} on S circle at A‴

$\Rightarrow y_{in} = 1.62 + j0.07$

$$Y_{in} = \frac{y_{in}}{Z_0} = 32.4 + j1.40 \times 10^{-3}\ \text{sec}$$

p.2
continued →

To travel from A to A' one must travel

$0.250\lambda - 0.144\lambda = 0.106\lambda$ → the shortest line length $= 0.106\lambda$

The voltage max occurs at

A' → $z = -0.106\lambda$

2.39

lossless line 50 Ω

(a) For short circuit $z_L = 0 + j0$ (A) at 0.00λ

Since a loss line repeats every $\lambda/2$ ⇒ 2.3λ towards generator $\equiv$ equivalent to 0.3λ toward the generator

↓

(B)

where $z_{in} = 0 - j3.07$

$\Rightarrow Z_{in} = z_{in} \times Z_o$

$= -j154\ \Omega$

(b) $Y_{in} = -j0.04 \Rightarrow y_{in} = -j2$ (C)

$y_{in} = Y_{in} Z_o$

→ D for z_{in}

$\overline{AD} = 0.074\lambda + n\frac{\lambda}{2}$

$n = 0, 1, 2, \ldots$

2-41

lossless line

100 Ω

length $= \frac{3\lambda}{8}$

$Z_{in} = -j2.5\,\Omega \longrightarrow z_{in} = -j0.025$ (A)

$Z_L = ?$

at 0.496 λ

and is 0.375 λ

toward the load

$$\Rightarrow \begin{array}{r} 0.496\lambda \\ -0.375\lambda \\ \hline 0.121\lambda \end{array}$$

Here:

(a) $z_L = 0 + j0.95$ $\rightsquigarrow$ (B)

$\longrightarrow Z_L = j95\,\Omega$

(b) open circuit line therefore at $z_L = \infty$ (C) $\Longrightarrow$

, which is at 0.250 λ

an open circuit line with $Z_{in} = j0.025$ must have a length of

$$\begin{array}{r} 0.250\lambda \\ -0.004\lambda \\ \hline 0.246\lambda \end{array}$$

2-42

lossless line 75 Ω

length $= 0.6\lambda$

$S = 1.8 \longrightarrow$ S circle must pass through $S = 1.8$

$\theta_r = -60°$

$|\Gamma|, Z_L, Z_{in} = ?$

circle of this radius

$$|\Gamma| = \frac{S-1}{S+1} = \frac{0.8}{2.8} = 0.285$$

(A)

interpection of circle of $|\Gamma|$ and the line of constant θ_r is at the load, Z_L, which has a value $z_L = 1.15 - j0.63$ → (B) at 0.3335λ

$$\hookrightarrow Z_L = z_L \underset{75\,\Omega}{Z_0} = 86.25 - j47.25\,\Omega$$

0.6λ toward the generator $\overset{\text{equiv.}}{\equiv}$ 0.1λ

$$\hookrightarrow 0.3335\lambda + 0.1\lambda = 0.4335\lambda \text{ at (C)}$$

where $z_{in} = 0.64 - j0.29$

$$\Rightarrow Z_{in} = 48 - j21.75\,\Omega$$

2-45

50 Ω lossline

$0.6\lambda \equiv 0.1\lambda$ equivalent

$Z_L = 50 + j25\,\Omega \rightarrow z_L = 1 + j0.5$ (A)

at $0.3\lambda \rightarrow R = 30\,\Omega$

$Z_{in} = ?$

$\left\{ \begin{array}{l} \text{at } Z'_L = 30\,\Omega \\ z'_L = 0.6 \end{array} \right.$

corresponding normalized load admittance at 0.394λ

need to get y_{in_L} due to z_L

parallel it with y'_L due to z'_L

get net $y'_{in} = y_{in_L} + y'_L$

$$z'_L = \frac{30\,\Omega}{50} = 0.6$$

$y'_L = 1.67$

$$\begin{array}{r} 0.1445\lambda \\ +\ 0.3\lambda \\ \hline 0.4445\lambda \end{array}$$

A → B for 0.3λ

z_{in_L} at (C)

$z_{in\,L}$ to $y_{in\,L}$ at (D)

$$\begin{array}{rr} y_{in\,L} = & 1.37 + j0.46 \\ + & 1.67 \\ \hline y'_{in} = & 3.04 + j0.46 \end{array}$$

at (E)

→ $Z_{in\,F} = 0.283 - j0.048$

Now go to CW 0.3λ to get z_{in}(overall)

$$\begin{array}{r} 0.4915\lambda \\ + 0.3\lambda \\ \hline 0.7915\lambda \\ - 0.5 \\ \hline 0.2915\lambda \end{array}$$

at G

then at H

$z_{in} = 1.95 - j1.38$

$Z_{in} = 97.5 - j69\ \Omega$

2-46

50 Ω lossless line

$Z_L = 75 - j20 \Rightarrow z_L = 1.5 - j0.4$ (A)

y_L at (B)

$$0.0425\lambda \rightarrow \begin{array}{r} 0.145\lambda \\ -0.0425\lambda \\ \hline 0.1025\lambda \end{array}$$

at (C): $y_{in} = 1 + j0.52$

↳ at (E) $-j0.52$

$$\Rightarrow \text{Length} = \begin{array}{r} 0.4238\lambda \\ 0.25\lambda \\ \hline 0.1738\lambda \end{array}$$

p.6

2-50

lossless line $\ell = 1\,m$

$Z_o = 50\,\Omega$

$u_p = \frac{2}{3}c$ $\quad Z_L = 25\,\Omega \Rightarrow u_p = 2\times10^8$ m/sec

$V_g = 60\,u(t)$ $\quad R_g = 100\,\Omega$ $\quad$ for 1m, $T = \frac{1}{2\times10^8} = 5$ nsec

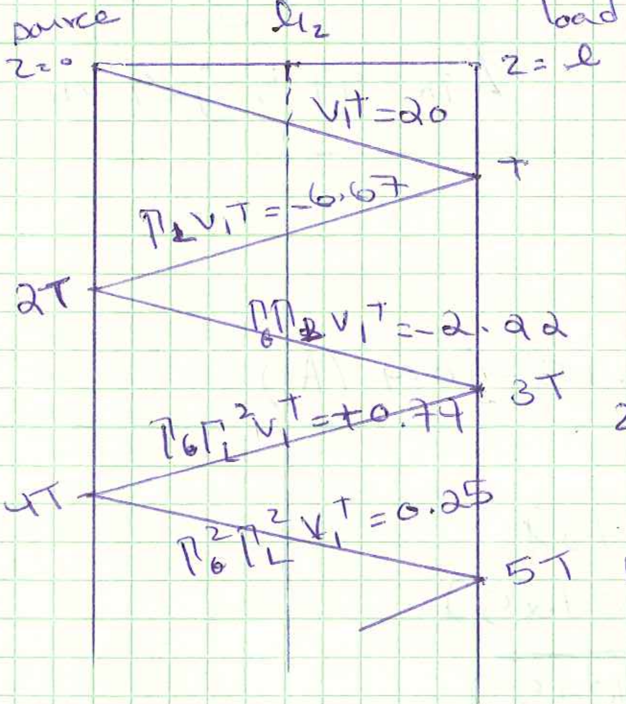
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{25-50}{25+50} = -\frac{25}{75} = -\frac{1}{3}$$

$$\Gamma_G = \frac{Z_G - Z_o}{Z_G + Z_o} = \frac{100-50}{100+50} = \frac{50}{150} = +\frac{1}{3}$$

25 nsec = 5T

$$V_1^+ = \frac{Z_o}{Z_o + Z_g} V_g = \frac{50}{150}\,60$$

$$\rightarrow V_1^+ = \frac{1}{3}(60) = 20$$



20 $\quad T/2^+ \rightarrow 3T/2^-$

-6.67 $\quad 3T/2^+ \rightarrow 5T/2^-$

13.33

-2.22 $\quad 5T/2^+ \rightarrow 7T/2^-$

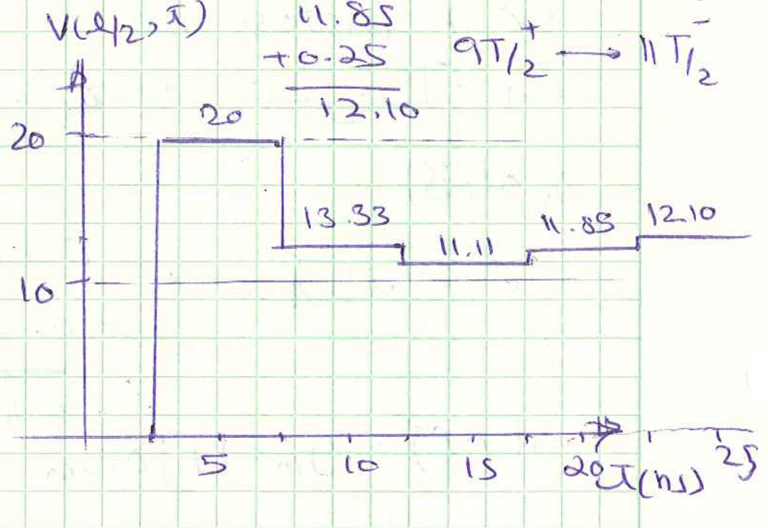
11.11

$+0.74$ $\quad 7T/2^+ \rightarrow 9T/2^-$

11.85

$+0.25$ $\quad 9T/2^+ \rightarrow 11T/2^-$

12.10

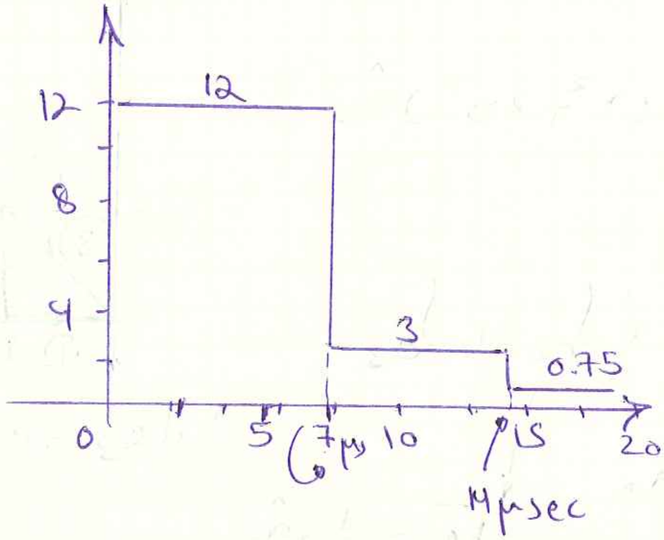


2-53

$Z_0 = 50\,\Omega$

$Z_L = 0$

$\epsilon_r = 4$



$$\frac{2l}{u_p} = 7\mu s$$

$$\longrightarrow u_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^{8}}{2} = 1.50\times10^{8}$$

$$l = \frac{(7\times10^{-6})(150\times10^{6})}{2} = 525\,m$$

$$V_1^+ = 12 = \frac{Z_0}{R_g+Z_0} V_g$$

$$\left.\begin{array}{l}\Gamma_L \Gamma_G V_1^+ = 3 \\ \Gamma_L^2 \Gamma_G V_1^+ = 0.75\end{array}\right\} \longrightarrow \Gamma_L \Gamma_G = \frac{3}{12} = \frac{1}{4}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0-50}{0+50} = -1 \Longrightarrow \Gamma_G = -\frac{1}{4}$$

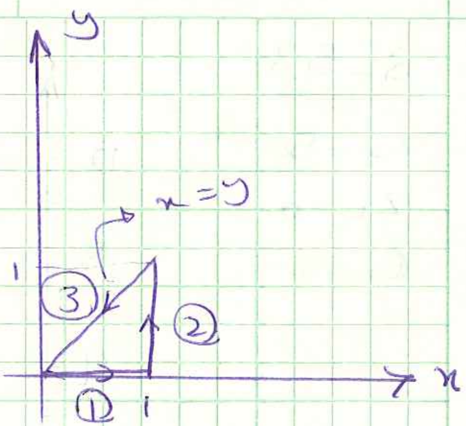
$$\Gamma_G = \frac{R_g - 50}{R_g + 50} = -\frac{1}{4} \Longrightarrow 5R_g = 150 \longrightarrow R_g = 30$$

$$12 = \left(\frac{50}{30+50}\right) V_g \Longrightarrow V_g = 19.2\,V$$

p.8

3.43

$E = xy\,\hat{a}_x - (x^2 + 2y^2)\,\hat{a}_y$



(a) $\oint \vec{E} \cdot \bar{dl} = ?$

$$\oint \bar{E} \cdot \bar{dl} = \int_{c_1} + \int_{c_2} + \int_{c_3}$$

$\bar{dl}_1 = dx\,\hat{a}_x$
$y = 0$

$\vec{dl}_2 = dy\,\hat{a}_y$
$x = 1$

$dl_3 = dx\,\hat{a}_x + dy\,\hat{a}_y$

$$\longrightarrow \int_{c_1} = \int_0^1 [(xy\,a_x - (x^2 + 2y^2)\,\hat{a}_y]\Big|_{y=0} \cdot dx\,\hat{a}_x = 0$$

$$\int_{c_2} = \int [xy\,\hat{a}_x - (1^2 + y(2)^2)] \cdot dy\,\hat{a}_y = \int_0^1 -(1 + 2y^2)\,dy$$

$$= -5/3$$

$$\int_{c_3} = \int [xy\,\hat{a}_x - (x^2 + 2y^2)\,\hat{a}_y] \cdot (dx\,\hat{a}_x + dy\,\hat{a}_y)$$

$$= \int_1^0 xy\Big|_{y=x} dx - \int_1^0 (x^2 + 2y^2)\Big|_{x=y} dy = +2/3$$

$$\longrightarrow \oint \bar{E} \cdot \bar{dl} = -5/3 + 0 + 2/3 = -1$$

(b) $\int_s (\bar{\nabla} \times \bar{E}) \cdot \bar{ds} = \oint \bar{E} \cdot \bar{dl}$ Stokes Theorem

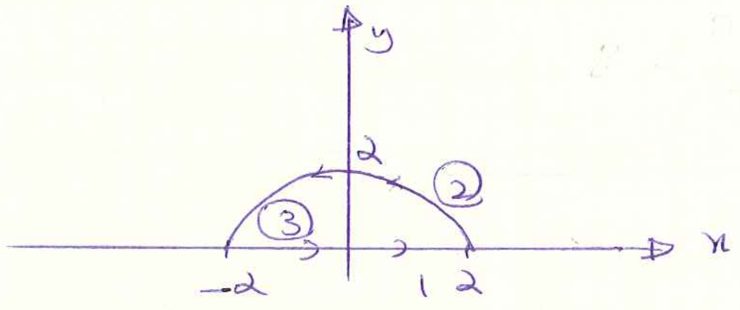
$ds = dx\,dy\,\hat{a}_z$

$$\bar{\nabla} \times \bar{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -3x\,\hat{a}_z$$

$$\longrightarrow \int (\bar{\nabla} \times \bar{E}) \cdot \bar{ds} = -3 \int_0^1 \int_0^x x\,dx\,dy = -1$$

3-45

$\vec{B} = r\cos\varphi\,\hat{a}_r + \sin\varphi\,\hat{a}_\varphi$



(a) $\int_c \vec{B}\cdot\vec{dl} = ?$

$\vec{dl}_1 = dr\,\hat{a}_r \quad \varphi = 0 \quad z = 0 \quad , r\Big|_0^2$

$\vec{dl}_2 = 2d\varphi\,\hat{a}_\varphi \; , z = 0 \; , \varphi\Big|_0^\pi$

$\vec{dl}_3 = dr\,\hat{a}_r \; , \varphi = \pi \; , z = 0 \; , r\Big|_{-2}^0$

$\rightarrow \vec{B}\cdot\vec{dl}_1 = r\cos\varphi\, dr\Big|_{\varphi=0}$

$\rightarrow \int_0^2 r\underbrace{\cos\cancel{\varphi}^{0}}_{1} dr = 2$

$\vec{B}\cdot\vec{dl}_2 = 2\sin\varphi\, d\varphi \rightarrow \int_2 = -2\cos\varphi\Big|_0^\pi = 4$

$\vec{B}\cdot\vec{dl}_3 = r\cos\varphi\, dr\Big|_{\varphi=\pi} \rightarrow \int_3 = -\frac{r^2}{2}\Big|_{-2}^0 = 2$

$\rightarrow \oint_c = 8$

(b) $\int_s (\vec{\nabla}\times\vec{B})\cdot\vec{ds}$

$\vec{\nabla}\times\vec{B} = \left(\frac{1}{r}\frac{\partial B_z}{\partial\varphi} - \frac{\partial B_\varphi}{\partial z}\right)\hat{a}_r + \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)\hat{a}_\varphi + \frac{1}{r}\left(\frac{\partial rB_r}{\partial r} - \frac{\partial B_r}{\partial\varphi}\right)\hat{a}_z$

$\rightarrow \quad = \frac{1}{r}(\sin\varphi \neq r\sin\varphi)\hat{a}_z$

$\vec{ds} = r\,dr\,d\varphi\,\hat{a}_z \qquad r\Big|_0^2 \quad \varphi\Big|_0^{2\pi}$

$\rightarrow \int = -4\cos\varphi\Big|_0^{\pi} = 8$

<u>3-49</u>

(a) $V = 4xy^2z^3 \rightarrow \nabla^2 V = 4(0 + 2xz^3 + 6xy^2z)$

(b) $V = xy + yz + zx \rightarrow \nabla^2 V = 0$

(c) $V = \frac{3}{x^2+y^2} \rightarrow \nabla^2 V = \frac{12}{(x^2+y^2)^2}$

↳ or $V = \frac{3}{r^2} \rightarrow \nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)$

$= \frac{12}{(r^2)^2}$

(d) $V = 5e^{-r}\cos\varphi$

↳ $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\varphi^2} + \overset{0}{\cancel{\frac{\partial^2 V}{\partial z^2}}}$

$= 5e^{-r}\cos\varphi\left(1 - \frac{1}{r} - \frac{1}{r^2}\right)$

(e) $V = 10e^{-R}\sin\theta$

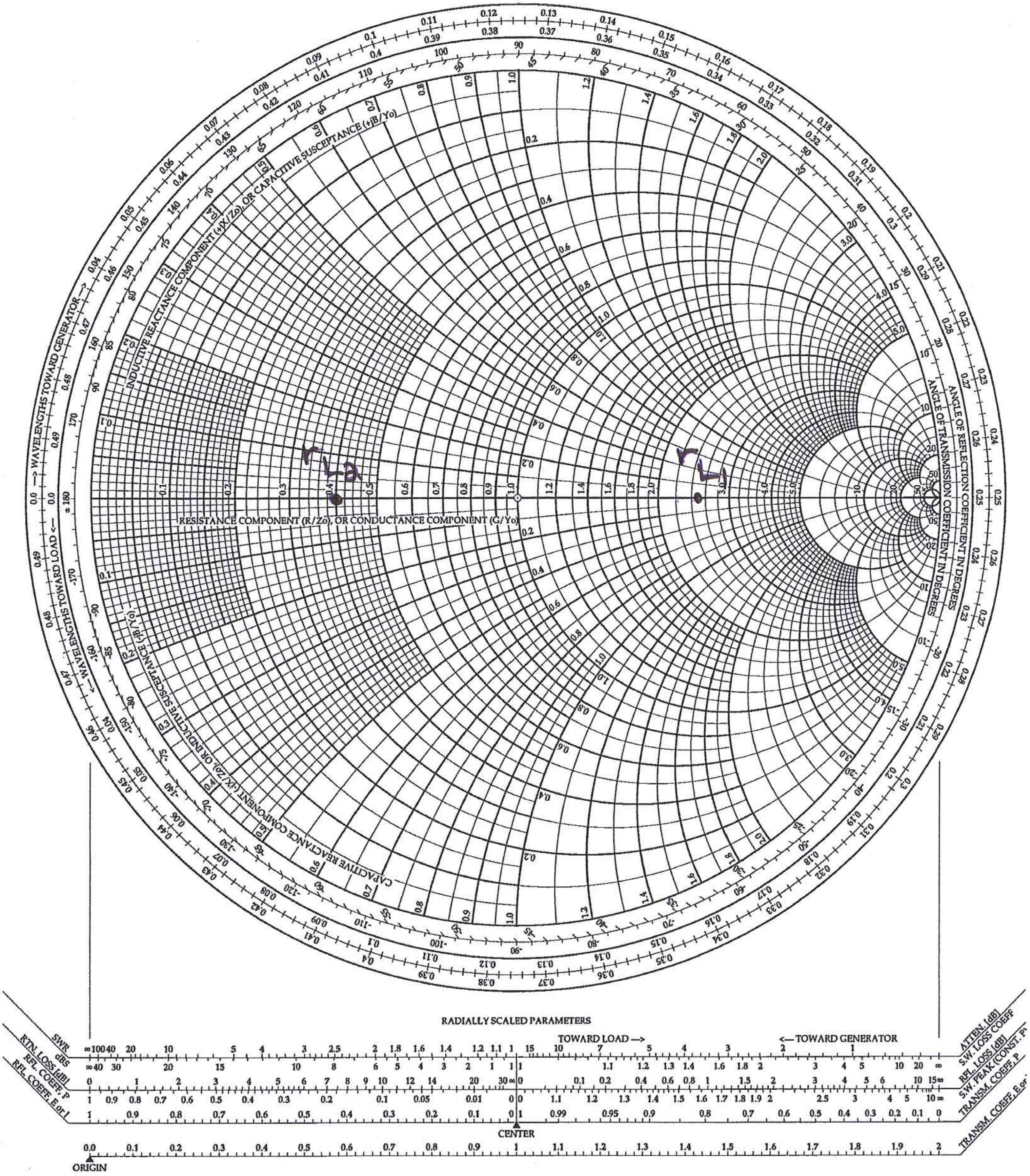
↳ $\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)$

$+ \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\varphi^2}$

$= 10e^{-R}\left[\left(1 - \frac{2}{R}\right)\sin\theta + \frac{\cos^2\theta - \sin^2\theta}{R^2\sin\theta}\right]$

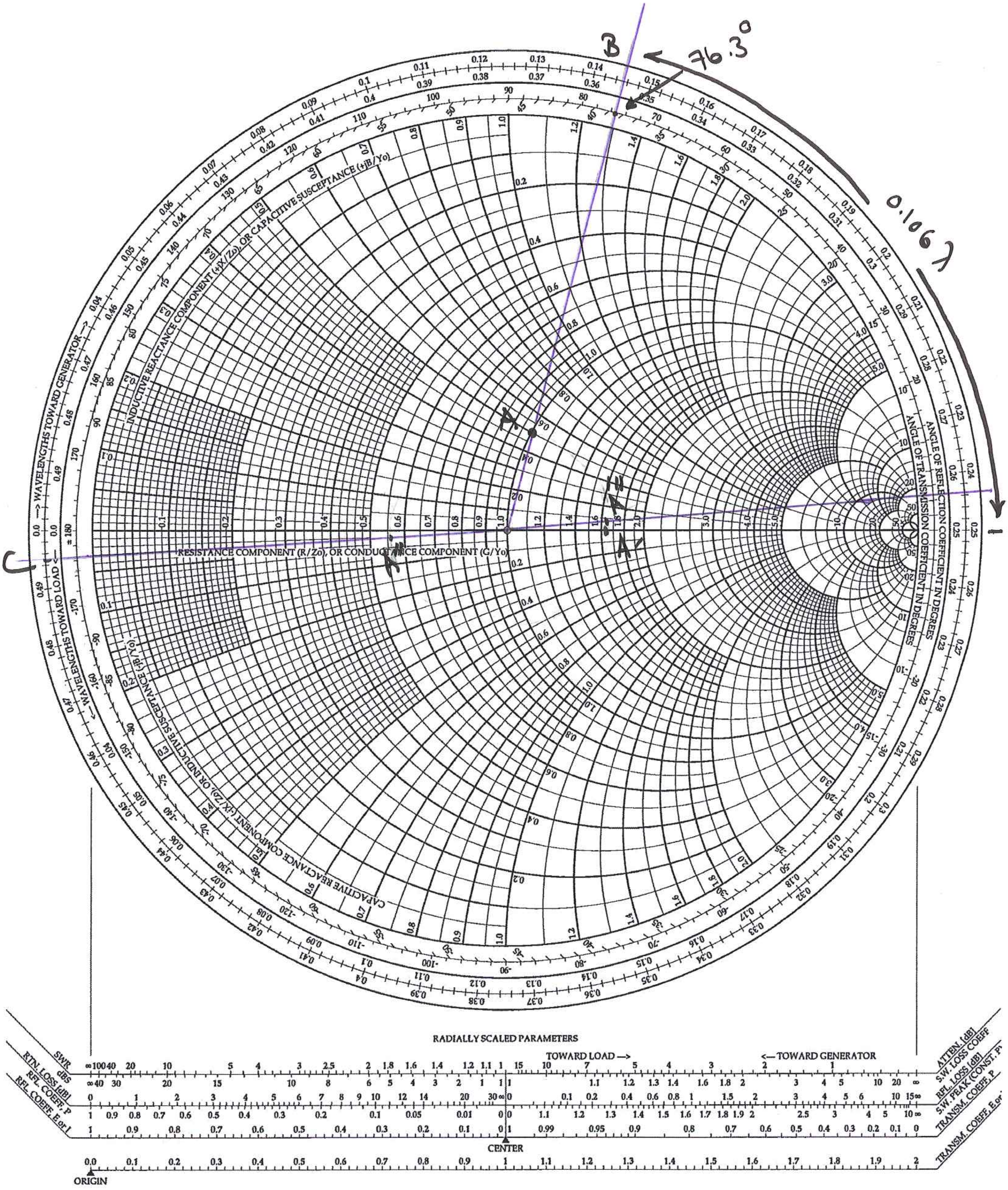
2-37

The Smith Chart



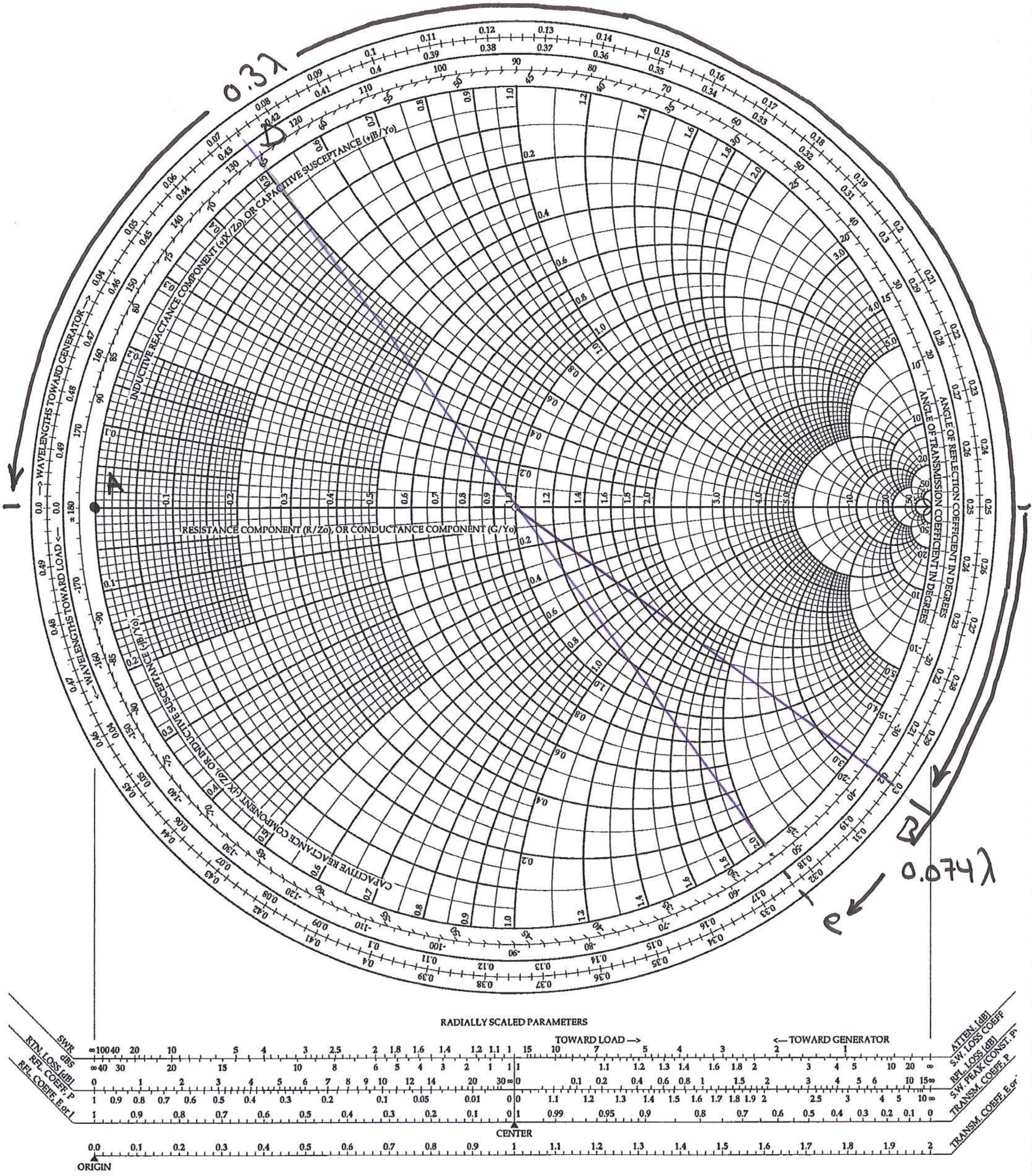
2-38

The Smith Chart



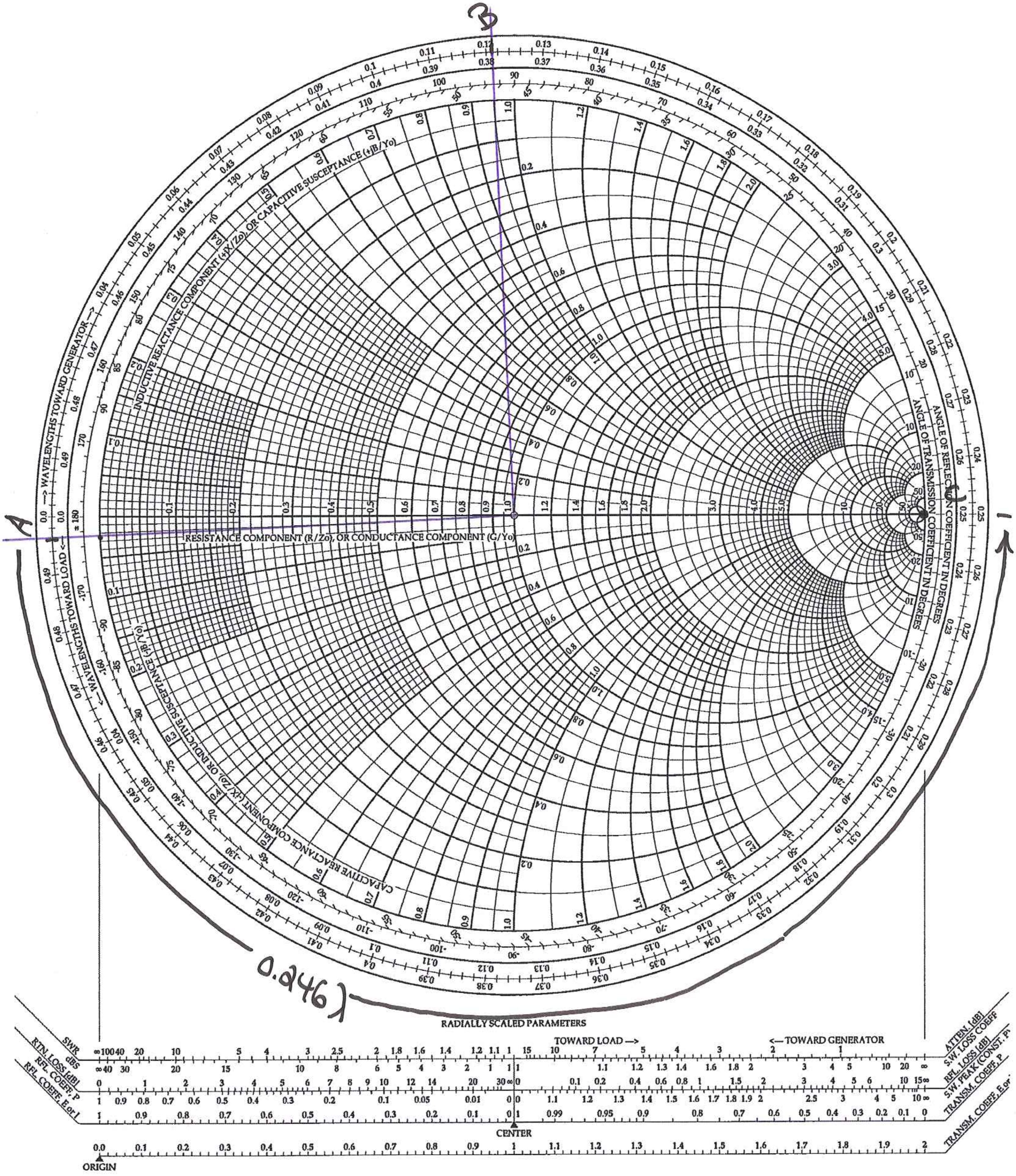
2-39

The Smith Chart



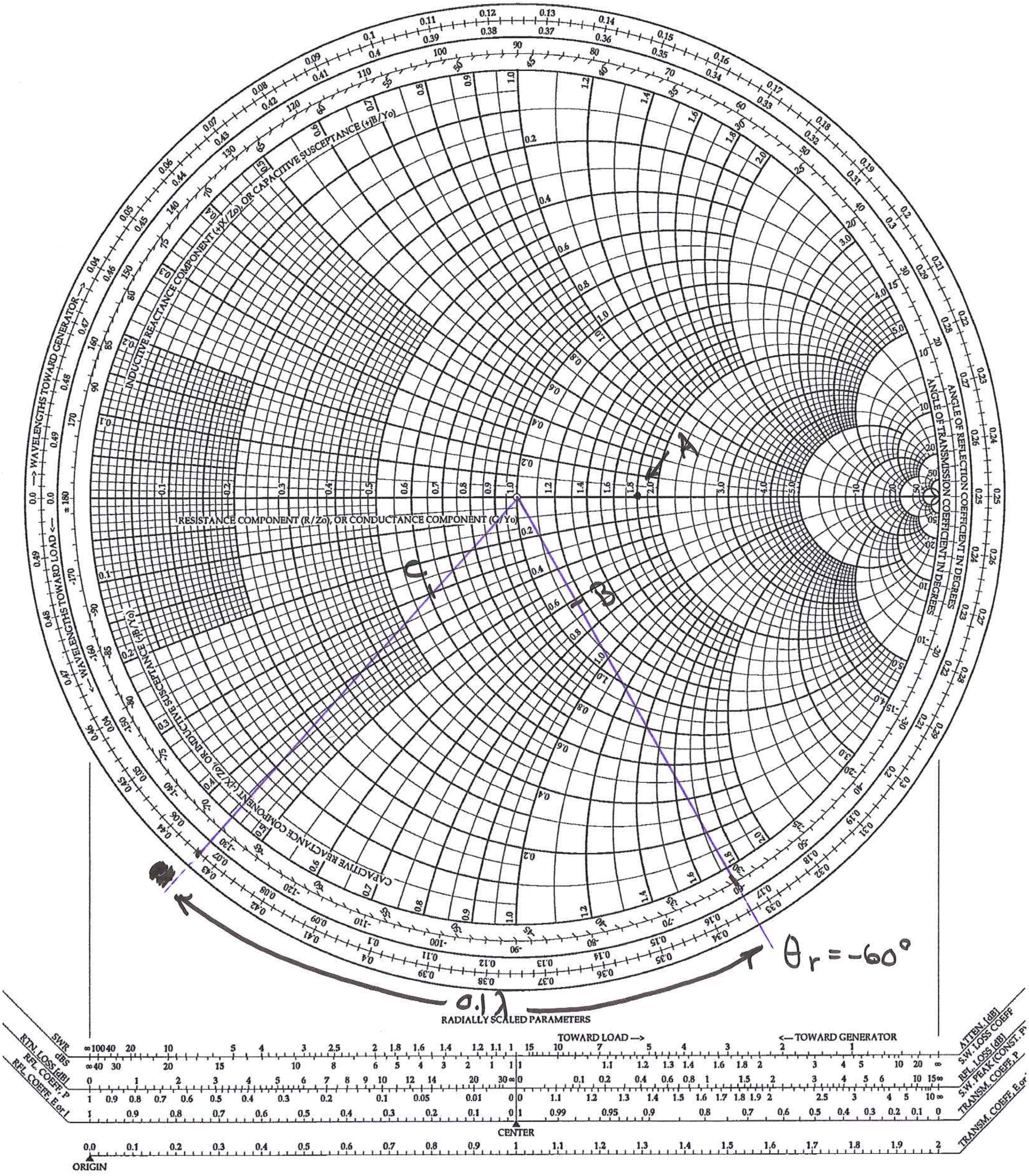
2-41

The Smith Chart



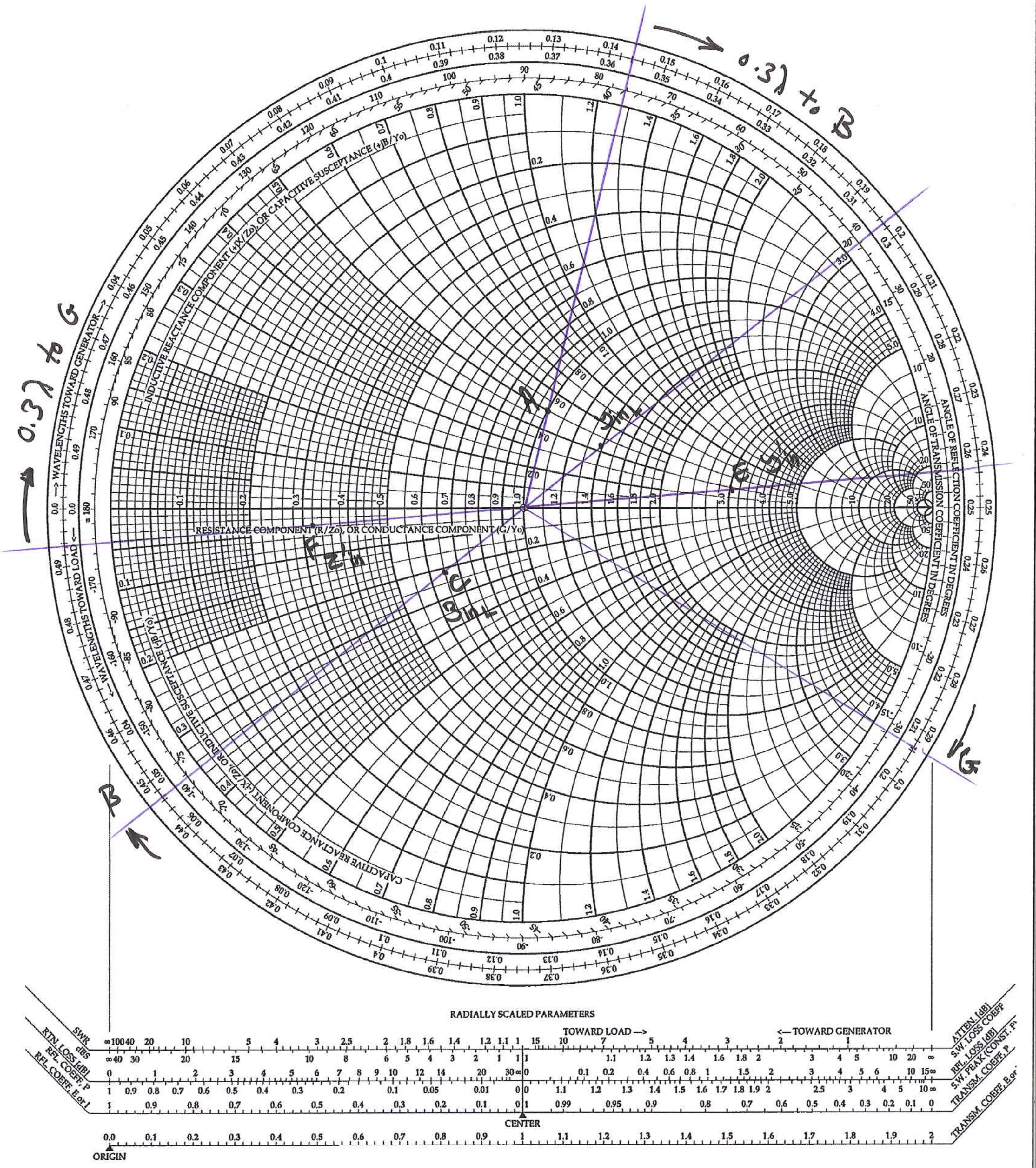
Q-42

The Smith Chart



2-45

The Smith Chart



2-46

The Smith Chart